

Tradeoffs in Metaprogramming

Todd Veldhuizen
Open Systems Laboratory
Indiana University Bloomington

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Two traditions of generics

- Safe (but restricted expressiveness)
 - ★ Alghard, CLU, ML, Haskell, Java, ...
 - ★ A parade of language features to recover expressiveness: parametric polymorphism, F-bounded polymorphism, type classes, typecase, ...
- Unsafe (but very expressive)
 - ★ EL1 [Wegbreit(1974), Holloway(1971)] — arbitrary expressions in type position, compiler had built-in partial evaluator for these.
 - ★ C++: traits (typecase), template metaprograms
 - ★ A parade of language features to increase safety: signatures, concepts

Can we have safe and expressive at the same time?

Metalinguages

A metalanguage is a special-purpose language for generating or transforming programs.

Stretch the definition to encompass languages for:

- Metaprogramming: YACC, TXL, Stratego, ...
- Code generation: SafeGen, MetaML, C++ Templates, ...
- Abstraction: Macros, generics, class definition syntax, ...
(A programming language is a pastiche of metalanguages.)

Metalanguages

(a) When is it possible to design metalanguages that...

- guarantee well-formedness?
- guarantee type safety?
- preserve semantics of the object language?
- always terminate?

and, (b) can we achieve the above without sacrificing expressive power?

You cannot have it both ways

Can I ...

1. Make C++ templates always halt without sacrificing expressive power?
2. Put a type system on JavaFront so that it only allows semantics-preserving transformations, but without sacrificing expressive power?
3. Design a metalanguage for specifying optimizations that permits any transformation that can be done in polynomial time, without making some transformations ridiculously hard to express?

Answer key: (1) No. (2) No. (3) No.

Tradeoffs

In designing a metalanguage, one must trade off various *facets*:

- Expressive power (the class of program behaviours that can be expressed)
- Safety properties
- Succinctness (do trivial metaprograms require vast amounts of code to express?)
- Decidable properties (what you can verify), computational complexity, etc. etc.

Why do we need special metalanguages?

In a universal language (Turing-complete), nontrivial properties are undecidable (Rice's theorem).

Cannot write a procedure that will decide whether a metaprogram

- emits only well-formed or typeable outputs;
- preserves semantics;
- terminates;
- runs in a given time or space bound (e.g., PTIME).

Capture

But sometimes we can find a programming language that “captures” a property.

Example. This is a highly undecidable property: (strict Σ_2^0):

$\text{Fin} \equiv$ Programs that terminate for at most a finite number of inputs.

Undecidable $\Rightarrow$ you can't write a procedure that decides whether a Java program satisfies the property.

Capture

But we can “capture” the property with a restricted language: only allow programs of the form:

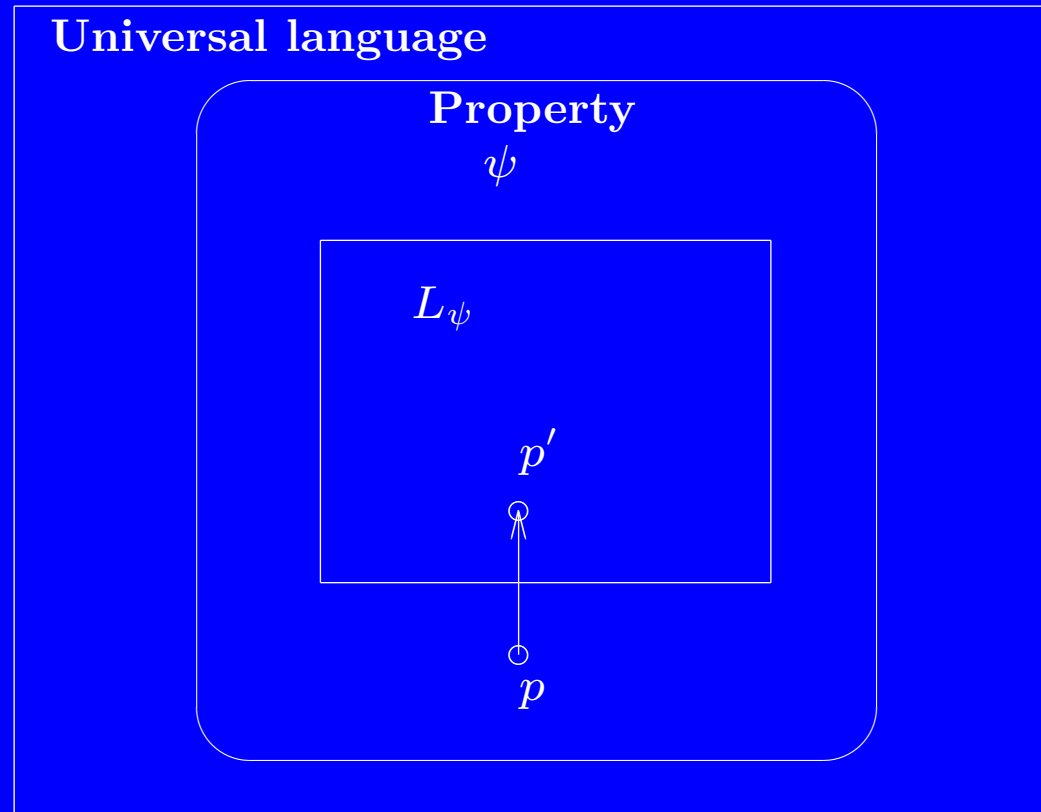
$$f(x) = \begin{cases} c_1 & \text{when } x = x_1 \\ c_2 & \text{when } x = x_2 \\ \vdots & \vdots \\ c_n & \text{when } x = x_n \\ \uparrow & \text{otherwise} \end{cases}$$

Every program in this sublanguage has the property Fin;
every behaviour in Fin has a representative in this sublanguage.

Capture

Say a restricted metalanguage *captures* a property ψ when:

1. Every program in the restricted language satisfies ψ ;
2. Every program (in a general-purpose language) that has the property ψ is equivalent to some program in the restricted language.



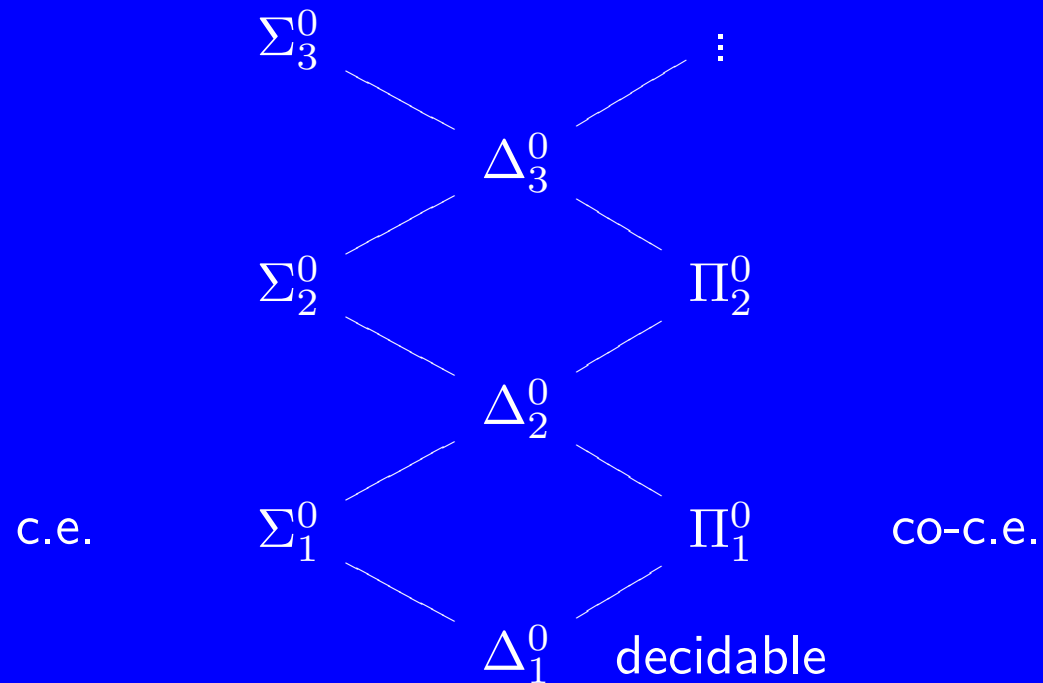
Metalanguages capturing properties

When can we find metalanguages capturing useful properties (well-formedness, typeable, semantics-preserving, ...)?

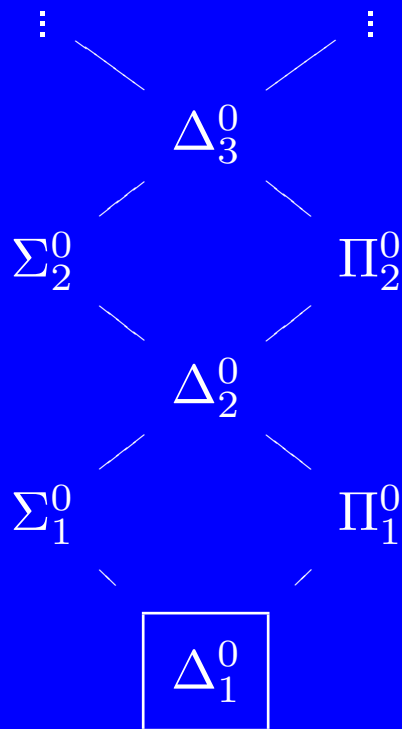
Computability theory is good at answering such questions.

The Arithmetical Hierarchy

Introduced by Kleene [Kleene(1950)] to classify noncomputable sets.



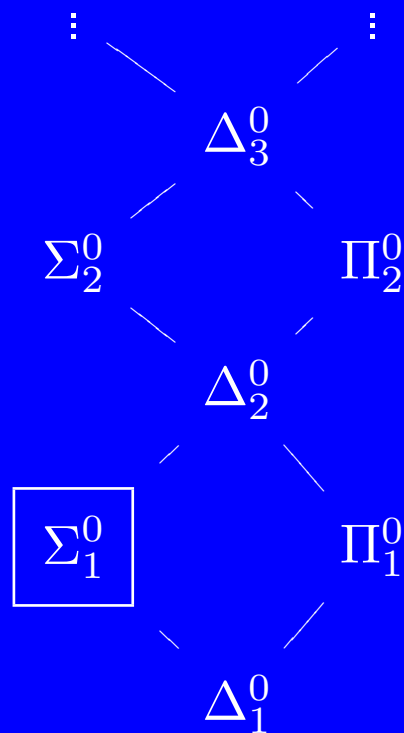
Class Δ_1^0



Decidable sets.

Rice's theorem: there are no nontrivial program properties in this class.

Class Σ_1^0



Computably enumerable (c.e.) sets (aka r.e.)

Sets with effective proof calculi

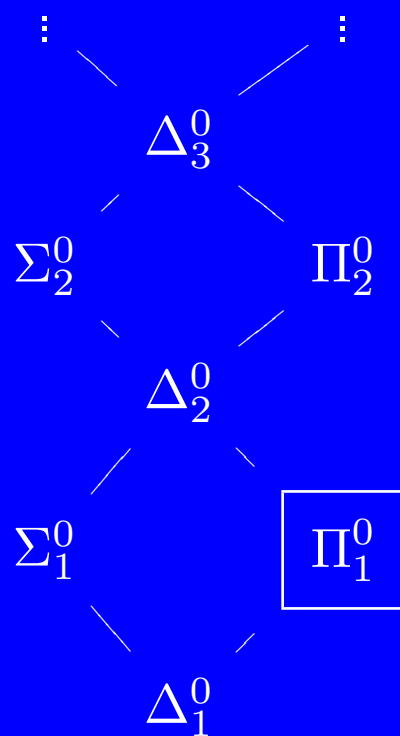
Sets with an effective inductive definition

Sets with a finite axiomatization

Properties living here: (not very interesting ones)

“Metaprogram halts for at least one input.”

Class Π_1^0



Co-Computably enumerable (co-c.e.) sets (aka r.e.)

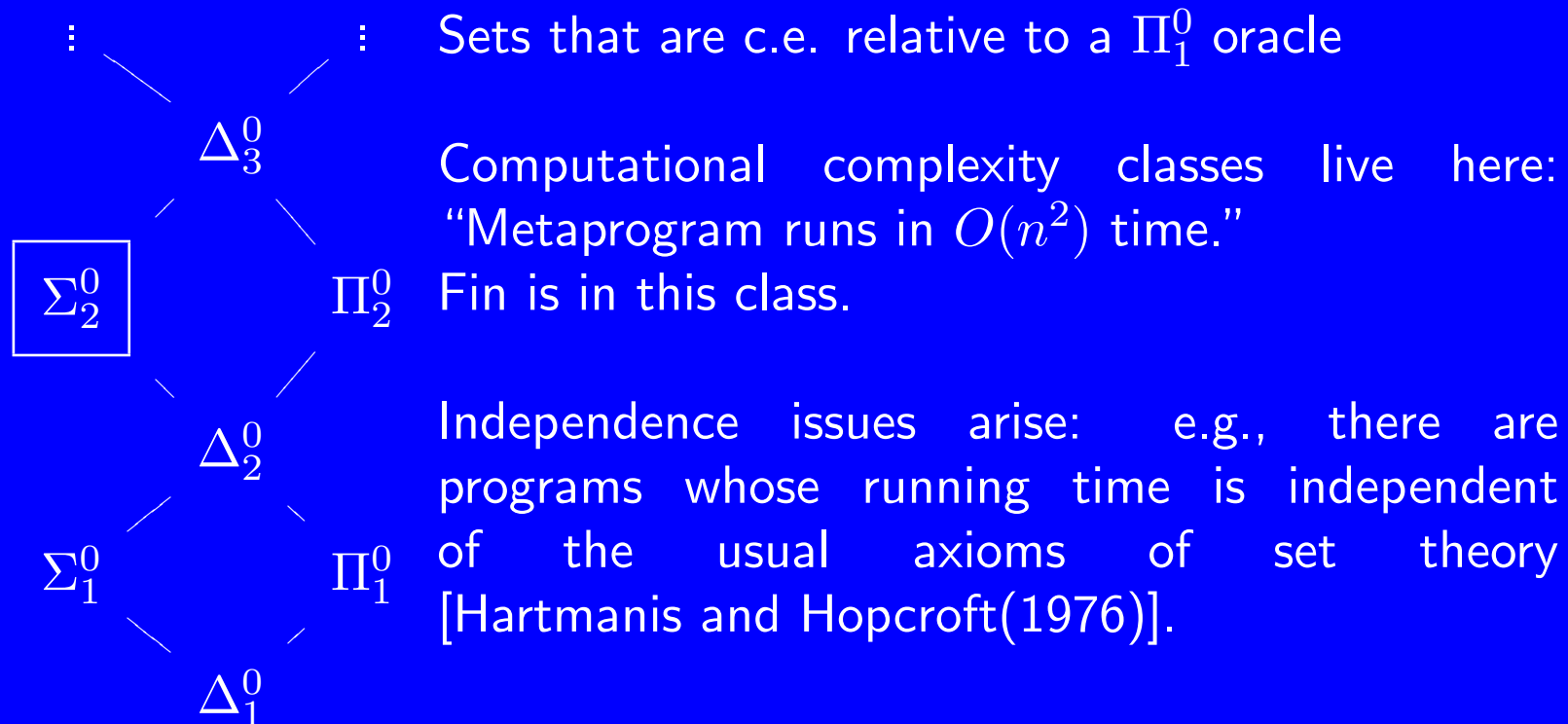
Sets with an effective coinductive definition

Properties living here:

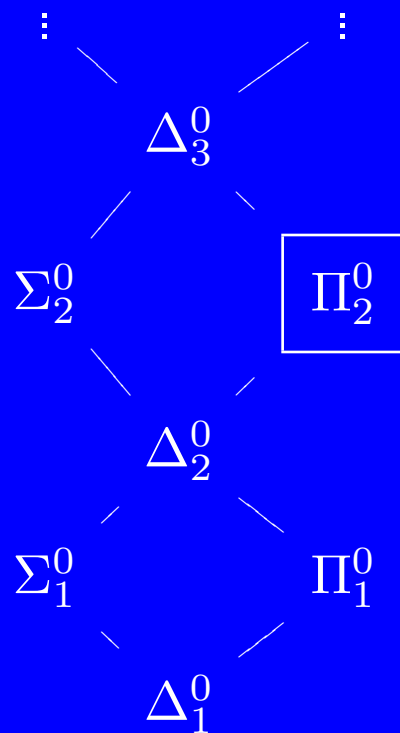
Runtime manageable properties (e.g., dynamic type safety).

Partial correctness properties: “If it halts, the metaprogram produces a well-formed/typeable instance.”

Class Σ_2^0



Class Π_2^0



Sets that are co-c.e. relative to a Σ_1^0 oracle

Properties living here:

“Metaprogram always halts and produces a well-formed/type-safe instance.”

“Metaprogram performs only semantics-preserving transformations.”

Succinctness

Sometimes when we translate programs into a restricted metalanguage, the size explodes. (e.g., DNF for boolean formulas: exponential blowup)

Sometimes this explosion in program length cannot be bounded by any computable function:

- e.g. restricted languages that are total (always terminate)
- Noncomputable blowup $\Rightarrow$ there are programs that require 10^{100} times more code to express in the restricted language.
- Whether we care is another matter (maybe they are not interesting programs).

Succinctness

(Defn) Succinct capture $\equiv$ capturing a property without noncomputable blowup in program size.

(Thm 6.3) It is impossible to succinctly capture properties not in Π_2^0 .

$\Rightarrow$ Languages capturing complexity classes (strict- Σ_2^0) have noncomputable blowup.

Silver lining: partial correctness properties are in $\Pi_1^0 \subset \Pi_2^0$.

Negative results on capture

There are no metalanguages capturing:

- (Prop 6.6) Metaprogram always halts.
- (Prop 6.7) Metaprogram always halts and produces a typeable/well-formed instance (total correctness).
- Metaprogram performs only semantics-preserving transformations.

Positive results on capture

There *is* a metalanguage capturing partial correctness:

- If the metaprogram halts, it produces a typesafe/well-formed instance

But we might not like it:

- Run the metaprogram on its input.
- Check the output. If it's bad, replace it with something safe.

i.e. no error messages.

Capture is tantamount to proof

L = a general-purpose language

L_ψ = a restricted language capturing ψ

(Thm 6.2) Transforming a program p from L into an equivalent program $p' \in L_\psi$ via semantics-preserving steps is equivalent to *proving* that $p \models \psi$.

(given proofs of ' L_ψ captures ψ ' and ' $p \sim p'$ ', we have a proof of $p \models \psi$.)

Corollary: If ψ is a nontrivial property, there can be no automated process that rewrites programs from L to L_ψ .

Heisenberg-like effects outside Σ_1^0

Σ_1^0 is the only class where we have finite axiomatizations $\equiv$ complete proof calculi.

Above this class we can only have partial axiomatizations (incomplete proof calculi).

Consequence: If ψ is a property not in Σ_1^0 , and L_ψ captures ψ , there will always be programs p that are equivalent to some $p' \in L_\psi$ but we cannot prove that $p \sim p'$.

Chasing properties with languages

We know that some properties cannot be captured (e.g., total correctness).

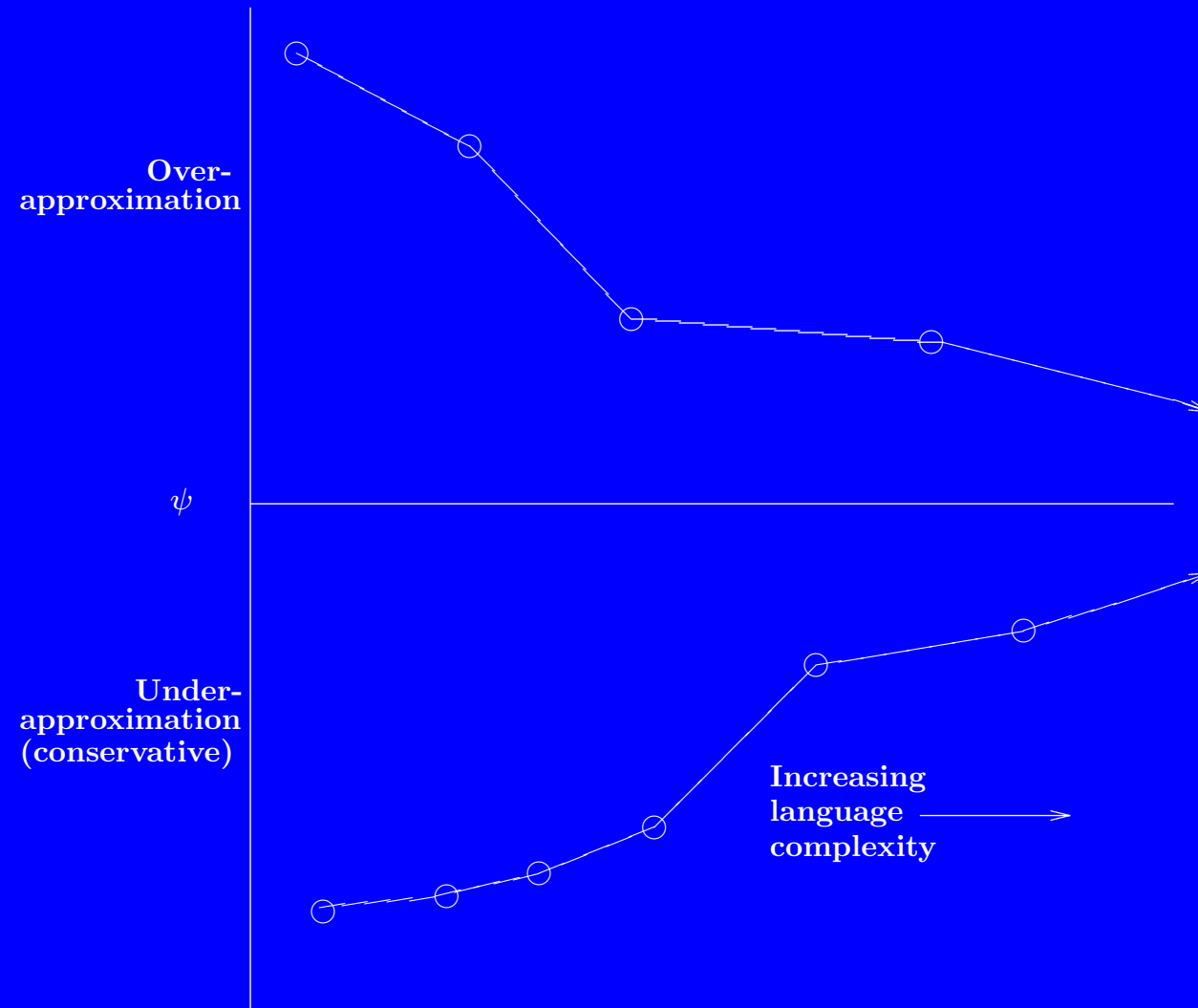
But, every 'functional' property is the limit of a sequence of languages with ever-increasing complexity:

$$L_0 \subset L_1 \subset L_2 \subset \dots$$

with $\lim_{i \rightarrow \infty} L_i = \psi$

and L_{i+1} requires a longer interpreter than L_i .

Two fundamentally opposed approaches to language design.



Conclusions

- Interesting properties of metaprograms are undecidable.
- But we can sometimes *capture* properties with restricted languages (e.g. partial correctness of metalanguages).
- If capture is not possible (e.g. total correctness), we can *chase* properties: a parade of language features, either
 - ★ Giving safety primacy, and recouping expressive power as language complexity $\rightarrow \infty$ (e.g., Haskell generics)
 - ★ Giving expressive power primacy, and recouping safety as language complexity $\rightarrow \infty$ (e.g., C++ generics)

Meta-conclusion

- Computability theory has useful explanatory power for tradeoffs in metalanguage design.

More details:

Todd L. Veldhuizen. Tradeoffs in Metaprogramming. ACM SIGPLAN Workshop on Partial Evaluation and Semantics-Based Program Manipulation (PEPM 2006), Charleston, South Carolina, January 9-10 2006.

Google "Tradeoffs in metaprogramming"

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