Chomsky Revisited

– or –

Everything you always wanted to know about *Parsing a Permutation*

Arthur Baars, Andres Löh and Doaitse Swierstra

Institute of Information and Computing Science
Utrecht University
e-mail: {arthurb,andres,doaitse}@cs.uu.nl

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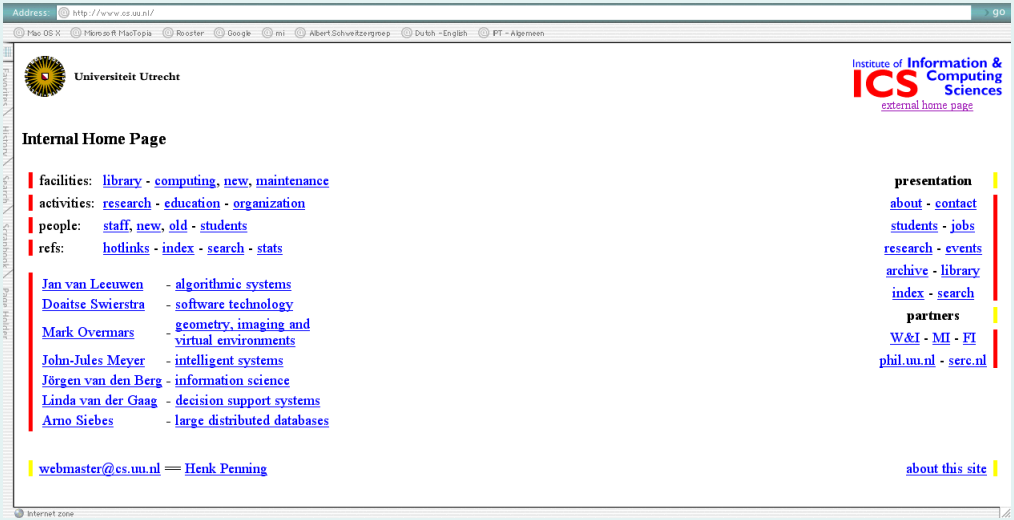
The Chomsky Hierarchy

The Chomsky hierarchy:

- Regular
- Context-free
- Context-sensitive
- Recursively enumerable

Example 1: HTML




```
<img width=190                  -- optional, integer
     height=54                  -- optional, integer
     src="/icons/ics.gif"       -- a string
     alt="HOME"                 -- a string
>
```

Attributes: order is irrelevant, each occurs exactly once

Example 2: BibTeX

```
@inProceedings
{ BaarsLoehSwierstra2001,
   author    = {Baars, Arthur and Loeh, Andres
                and Swierstra, S. Doaitse},
   title     = {Parsing Permutation Phrases},
   booktitle = {Preliminary proceedings of
                Haskell workshop 2001,
                # techreport # UU-CS-2001-23},
   year      = 2001,
   pages     = {171--182},
   editor    = {Hinze, Ralf},
}
```

Fields: order is irrelevant, each occurs exactly once

Approach 1

$Attributes \quad ::= (Name \text{ '='} \ Value)*$

Step 1(context-free):

- Parse input as a sequence of assignments

Approach 1

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Step 1(context-free):

- Parse input as a sequence of assignments

Step 2(semantic action):

- Check for valid names
- Check whether each name occurs only once
- Check whether each value has the right type

Approach 2

$$
\begin{array}{ll}
\mathit{Attributes} & ::= \mathit{Field}* \\
\mathit{Field} & ::= \mathit{Width} \mid \mathit{Height} \mid \mathit{Src} \mid \mathit{Alt}
\end{array}
$$

$$
\begin{array}{ll}
\mathit{Width} & ::= \texttt{"width="}\ \mathit{Number} \\
\mathit{Height} & ::= \texttt{"height="}\ \mathit{Number} \\
\mathit{Src} & ::= \texttt{"src="}\ \mathit{String} \\
\mathit{Alt} & ::= \texttt{"alt="}\ \mathit{String}
\end{array}
$$

- Name and type checks moved to syntactic level
- Occurrence check still in second step

Approach 3

$$
\begin{array}{lll}
\mathit{Attributes} & ::= & \mathit{Width}\ \mathit{Height}\ \mathit{Src}\ \mathit{Alt} \\
 & | & \mathit{Width}\ \mathit{Height}\ \mathit{Alt}\ \mathit{Src} \\
 & | & \mathit{Width}\ \mathit{Src}\ \mathit{Height}\ \mathit{Alt} \\
 & | & \mathit{Width}\ \mathit{Src}\ \mathit{Alt}\ \mathit{Height} \\
 & | & \mathit{Width}\ \mathit{Alt}\ \mathit{Height}\ \mathit{Src} \\
 & | & \mathit{Width}\ \mathit{Alt}\ \mathit{Src}\ \mathit{Height} \\
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 & | & \mathit{Src}\ \mathit{Width}\ \mathit{Height}\ \mathit{Alt} \\
 & | & \mathit{Src}\ \mathit{Width}\ \mathit{Alt}\ \mathit{Height} \\
 & | & \mathit{Src}\ \mathit{Height}\ \mathit{Width}\ \mathit{Alt} \\
 & | & \ldots
\end{array}
$$

EBNF grammars grow too large

BNF:

- alternative "|"
- composition "␣"
- terminals "'s'"
- empty string ϵ

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- Formalism with basic operators $\Longleftarrow$ **Library, Package**
- Extension mechanism in the formalism $\Longleftarrow$ **Programming language**

Combinator language

- Basic operators "combine" entities from a formalism
- Extension mechanism is borrowed from a programming language

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- Basic $\underbrace{\text{operators}}_{|,*,?,\epsilon}$ "combine" $\underbrace{\text{entities}}_{parsers}$ from a $\underbrace{\text{formalism}}_{EBNF}$
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- Basic $\underbrace{\text{operators}}_{|,*,?,\epsilon}$ "combine" $\underbrace{\text{entities}}_{parsers}$ from a $\underbrace{\text{formalism}}_{EBNF}$
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A Parser Combinator Library

Basic operators:

	EBNF	Haskell
alternative	$a \mid b$	
composition	$a\ b$	
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A Parser Combinator Library

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A Parser Combinator Library

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composition	$a\ b$	a <*> b
terminals	's'	$symbol$'s'
empty string	ϵ	

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empty string	ϵ	$epsilon$

A Parser Combinator Library

Basic operators:

	EBNF	Haskell
alternative	$a \mid b$	a `<|>` b
composition	$a\ b$	a `<*>` b
terminals	`'s'`	*symbol* `'s'`
empty string	ϵ	*epsilon*

Other operators:

repetition	$a*$	*many* a
option	$a?$	*option* a
grouping	$(\ldots)$	$(\ldots)$

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Example of use:

```
symbol 'H'<*>
symbol 'o'<*>
symbol 'i'
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symbol 'i'
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composition	$a\ b$	a `<*>` b
terminals	`'s'`	*symbol* `'s'`
empty string	ϵ	*epsilon*

Other operators:

repetition	$a*$	*many* a
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grouping	$(\ldots)$	$(\ldots)$

Example of use:

```
many (symbol 'H'<*>
      option (symbol 'o')<*>
      symbol 'i'
     )
```

What is a Parser

$String \longrightarrow a$

What is a Parser

$String \longrightarrow [a]$

What is a Parser

$$String \rightarrow [(a, String)]$$

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$$String \rightarrow [(a, String)]$$

Abbreviation:

$$Parser\ a$$

Types of the Combinators

<|>	$:: Parser\ a \longrightarrow Parser\ a \longrightarrow Parser\ a$

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<|>	$:: Parser\ a \rightarrow Parser\ a \rightarrow Parser\ a$

<*>	$:: Parser\ a \rightarrow Parser\ b \rightarrow Parser\ (a, b)$

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<|>	$:: Parser\ a \to Parser\ a \to Parser\ a$
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<*>	$:: Parser\ (b \to c) \to Parser\ b \to Parser\ c$
symbol	$:: Char \to Parser\ Char$

Types of the Combinators

`<|>`	$:: Parser\ a \rightarrow Parser\ a \rightarrow Parser\ a$
`<*>`	$:: Parser\ a \rightarrow Parser\ b \rightarrow Parser\ (a, b)$
`<*>`	$:: Parser\ (b \rightarrow c) \rightarrow Parser\ b \rightarrow Parser\ c$
symbol	$:: Char \rightarrow Parser\ Char$
epsilon	$:: a \rightarrow Parser\ a$

Types of the Combinators

`<|>`	$:: Parser\ a \rightarrow Parser\ a \rightarrow Parser\ a$
`<*>`	$:: Parser\ a \rightarrow Parser\ b \rightarrow Parser\ (a, b)$
`<*>`	$:: Parser\ (b \rightarrow c) \rightarrow Parser\ b \rightarrow Parser\ c$
symbol	$:: Char \rightarrow Parser\ Char$
epsilon	$:: a \rightarrow Parser\ a$
`<$>`	$:: (a \rightarrow b) \rightarrow Parser\ a \rightarrow Parser\ b$

Make your own extensions

EBNF:

$$
\begin{aligned}
&\mathit{many} && :: \quad \mathit{Parser}\ a \rightarrow \mathit{Parser}\ [a] \\
&\mathit{many}\ x && = \quad x\ \texttt{<*>}\ \mathit{many}\ x \\
& && \texttt{<|>}\ \mathit{epsilon} \\
&\mathit{option}\ x && = \ldots
\end{aligned}
$$

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$$
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& && \texttt{<|>}\ epsilon\ [\,] \\
&option\ x && = \ldots
\end{aligned}
$$

EEBNF:

$$
\begin{aligned}
&token && :: [Char] \rightarrow Parser\ [Char] \\
&token\ (s : rest) && = symbol\ s\ \texttt{<*>}\ token\ rest \\
&token\ [\,] && = epsilon
\end{aligned}
$$

Make your own extensions

EBNF:

```
many       :: Parser a → Parser [a]
many x     = (:) <$> x <*> many x
           <|> epsilon []
option x   = ...
```

EEBNF:

```
token      :: [Char] → Parser [Char]
token (s : rest) = (:) <$> symbol s <*> token rest
token []   = epsilon ""
```

Make your own extensions

EBNF:

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many       :: Parser a → Parser [a]
many x     = (:) <$> x <*> many x
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EEBNF:

```
token      :: [Char] → Parser [Char]
token (s : rest) = (:) <$> symbol s <*> token rest
token []   = epsilon ""
```

EEEBNF:

```
permute    = ...
<||>       = ...
```

Example: IMG-tag attributes

Parser for attributes:

```
permute (field "width" integer
        <||>field "height" integer
        <||>field "src" string
        <||>field "alt" string
        )
```

Helper function for parsing assignments:

```
field name val = glue<$>token name <*> symbol '=' <*> val
  where glue _ _ v = v
```

Implementation idea

$$s = permute\ (a \texttt{<|>}\ b \texttt{<|>}\ c)$$

should be equivalent to:

$$\begin{aligned} s = {} & a \texttt{<*>}\ b \texttt{<*>}\ c \\ & \texttt{<|>} a \texttt{<*>}\ c \texttt{<*>}\ b \\ & \texttt{<|>} b \texttt{<*>}\ a \texttt{<*>}\ c \\ & \texttt{<|>} b \texttt{<*>}\ c \texttt{<*>}\ a \\ & \texttt{<|>} c \texttt{<*>}\ a \texttt{<*>}\ b \\ & \texttt{<|>} c \texttt{<*>}\ b \texttt{<*>}\ a \end{aligned}$$

Implementation idea

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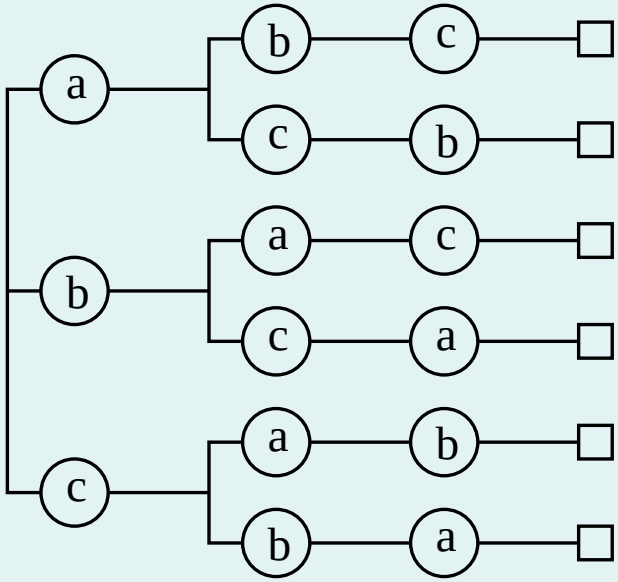
or, with left-factorization:

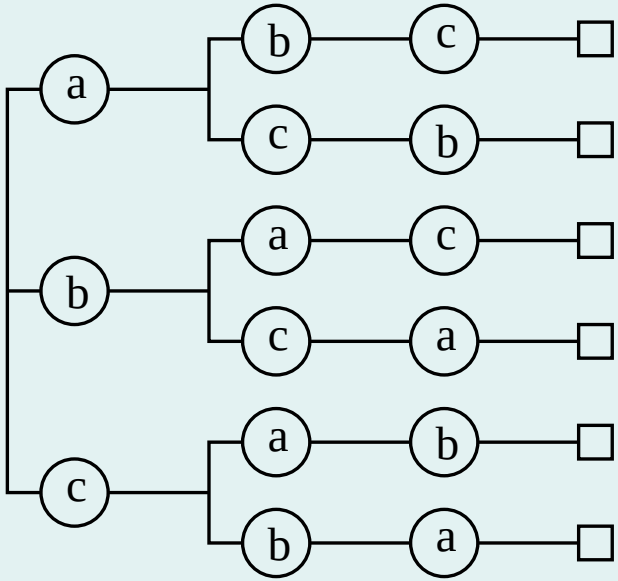
$$\begin{aligned} s = {} & a \mathbin{<*>} (b \mathbin{<*>} c \mathbin{<|>} c \mathbin{<*>} b) \\ \mathbin{<|>} & b \mathbin{<*>} (a \mathbin{<*>} c \mathbin{<|>} c \mathbin{<*>} a) \\ \mathbin{<|>} & c \mathbin{<*>} (a \mathbin{<*>} b \mathbin{<|>} b \mathbin{<*>} a) \end{aligned}$$

Implementation idea



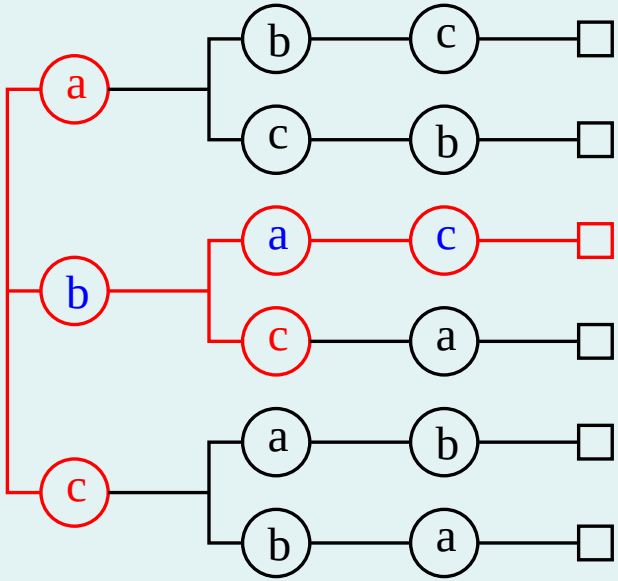
- <| |>: constructs an intermediate data structure(decision tree)
- *permute*: transforms it into a parser

Complexity



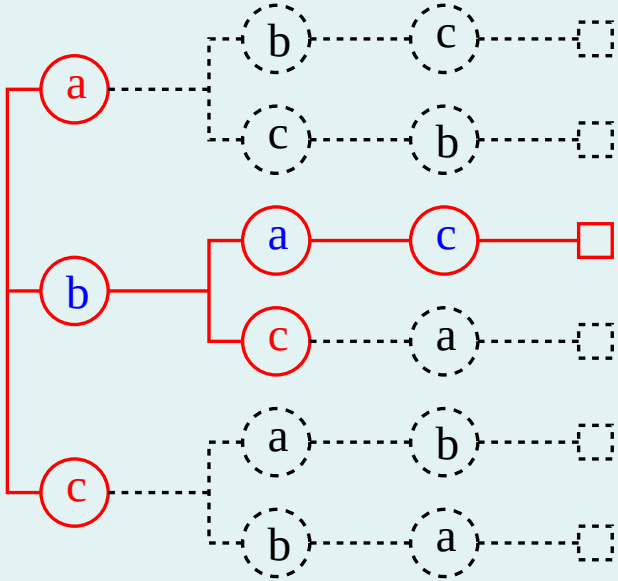
Size of the intermediate structure: O(n!)

Complexity



Example: parse string "bac"

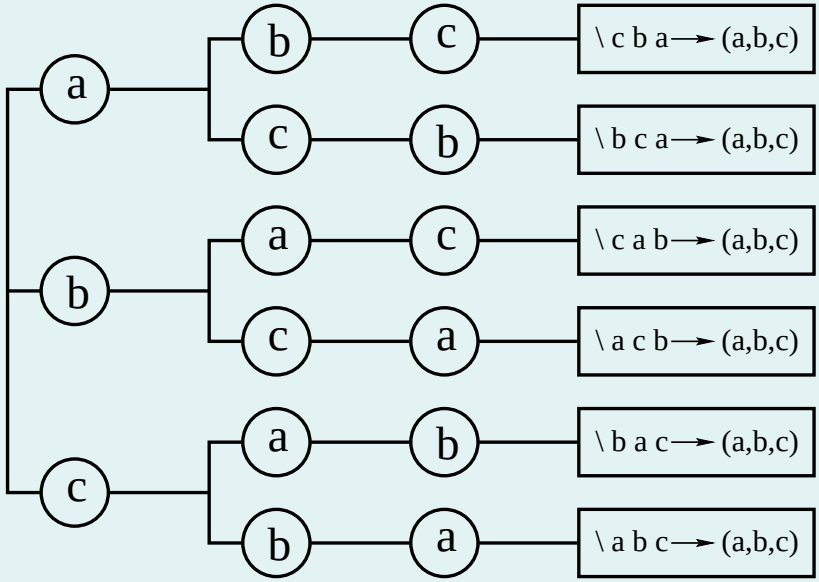
Complexity



Thanks to lazy evaluation only one path is constructed

Size of the intermediate structure: $O(n^2)$

Finishing Touch: Reordering



Each leaf contains a special reordering function

Recap

- <| |>: combinator to construct large permutation grammar
  - Provides notation for writing such grammars easily
  - Makes programmer happy
- *permute*: uses grammar to parse input
- Lazy evaluation:
  - Only small part of the grammar is computed
  - Makes computer happy

Intermediate data structure (1)

Structure of the tree:

$$
\begin{array}{lll}
\textbf{data}\ \mathit{Tree} & = & \mathit{Choice}\ [\mathit{Branch}] \\
 & | & \mathit{Leaf} \\
\textbf{data}\ \mathit{Branch} & = & \mathit{Branch}\ (\mathit{Parser?})\ \mathit{Tree}
\end{array}
$$

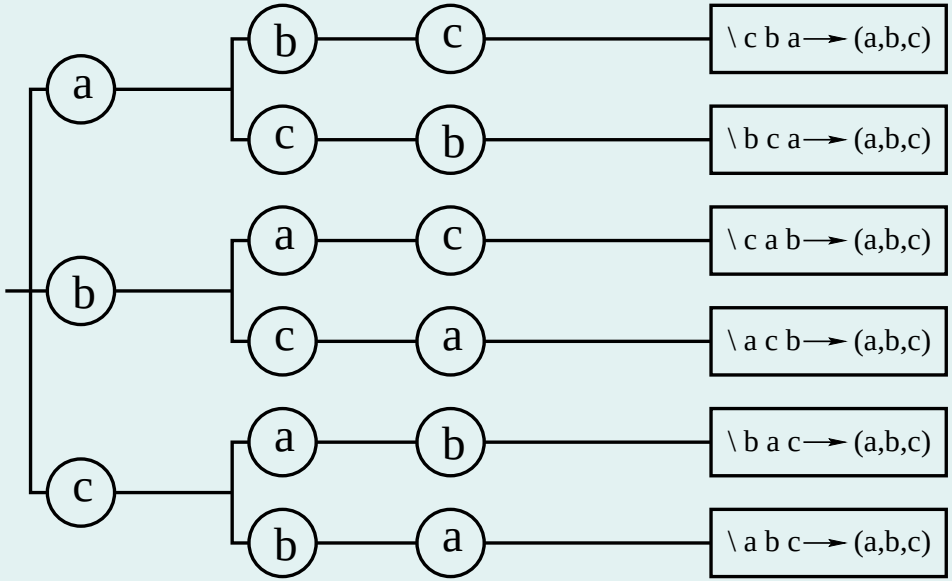
Intermediate data structure (2)

Structure of the tree:

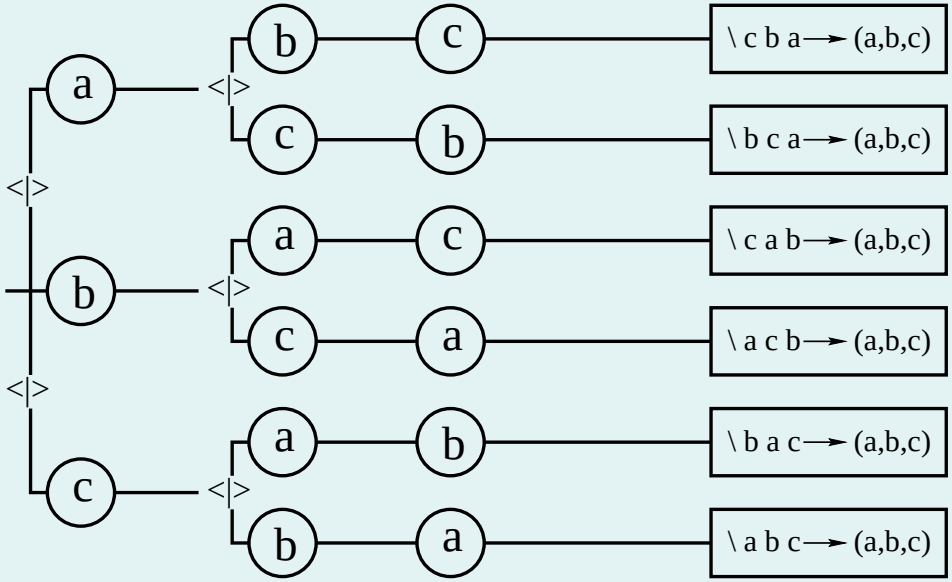
$$
\begin{array}{lcl}
\textbf{data}\ \mathit{Tree}\ a & = & \mathit{Choice}\ [\mathit{Branch}\ a] \\
 & | & \mathit{Leaf}\ a \\
\textbf{data}\ \mathit{Branch}\ a & = & \exists x.\mathit{Branch}\ (\mathit{Parser}\ x)\ (\mathit{Tree}\ (x \to a))
\end{array}
$$

an element of type a is constructed by applying the result of the rest of the path (of type $x \to a$) to the result of the parser in the branch (of type x)!

Transforming intermediate structure



Transforming intermediate structure



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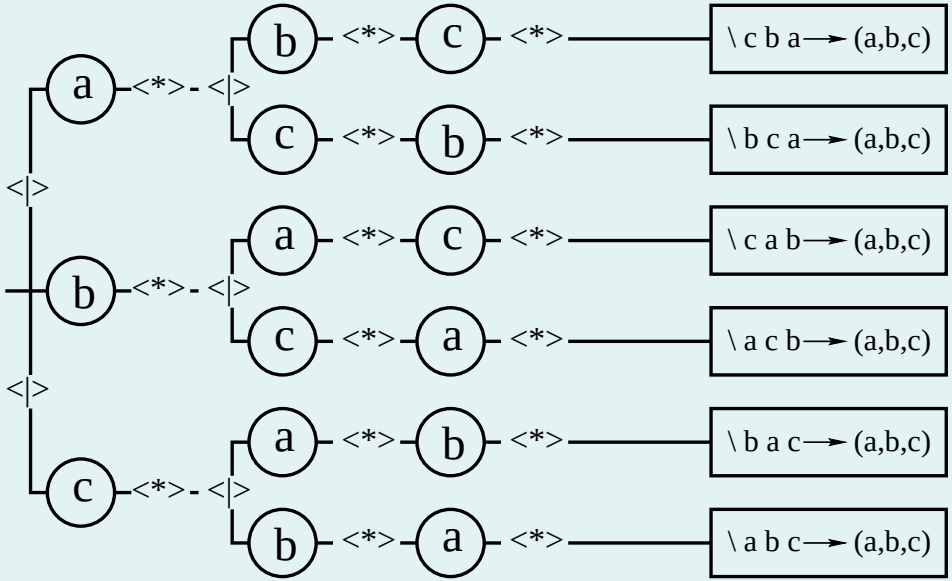

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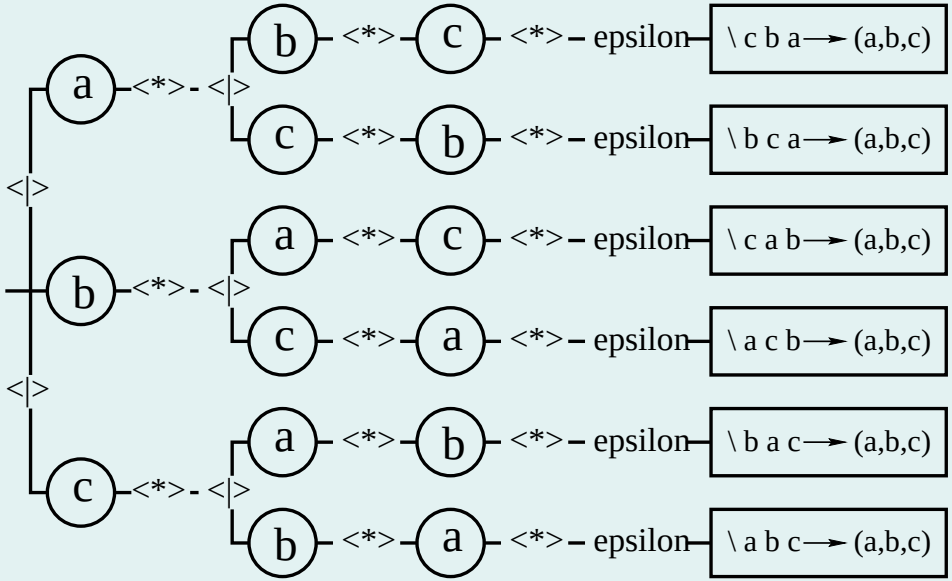

Transforming intermediate structure

permute traverses the grammar and constructs a parser:

```
permute (Leaf f)      = epsilon f
permute (Choice bs)   = choice [glue <$> p <*> permute rest
                                | Branch p rest ← bs
                               ]
  where glue x f = f x
```

The function *choice* takes a list of parsers and puts <|> between them

Summary

- Permutation parsing can be done context-free
- Necessary large grammars can be generated
  (by <|| > and $permute$ combinators)
- Lazy evaluation reduces complexity from O(n!) to O(n^2)

Conclusions

Combinators are cool

- Formalisms are programs
- Extendible
- Executable

Haskell is hot

- Higher order functions
- Lazy evaluation
- User defined operators
- ...