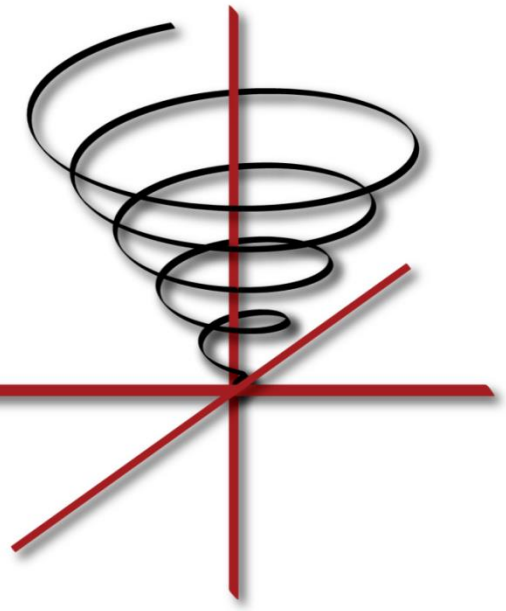
High Assurance Spiral:
Co-Synthesizing Proof and Implementation from High-Level Specification



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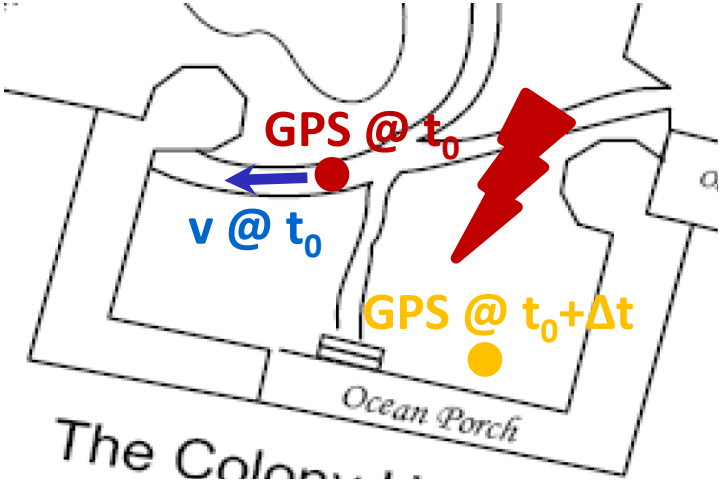
Motivation: Model-Based High Assurance

Multi-sensor ground vehicles

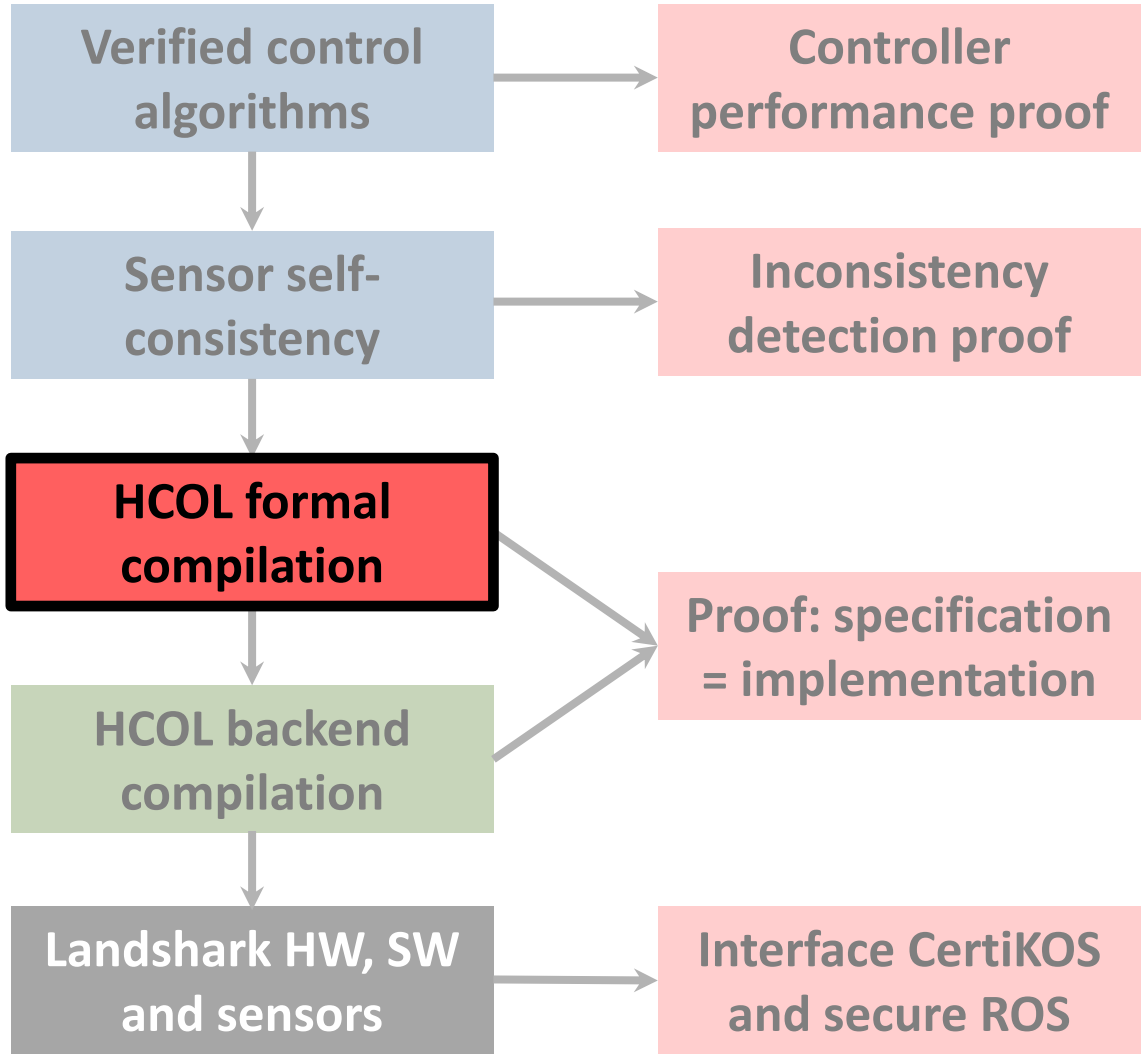
- **Multiple sensors:** GPS, compass, accelerometer, IMU, etc.
- **Control:** waypoints, joystick vector
- **Vehicle model:** laws of physics, vehicle state
- **Map data:** Terrain, possible paths, obstacles

Assurance Through Consistency

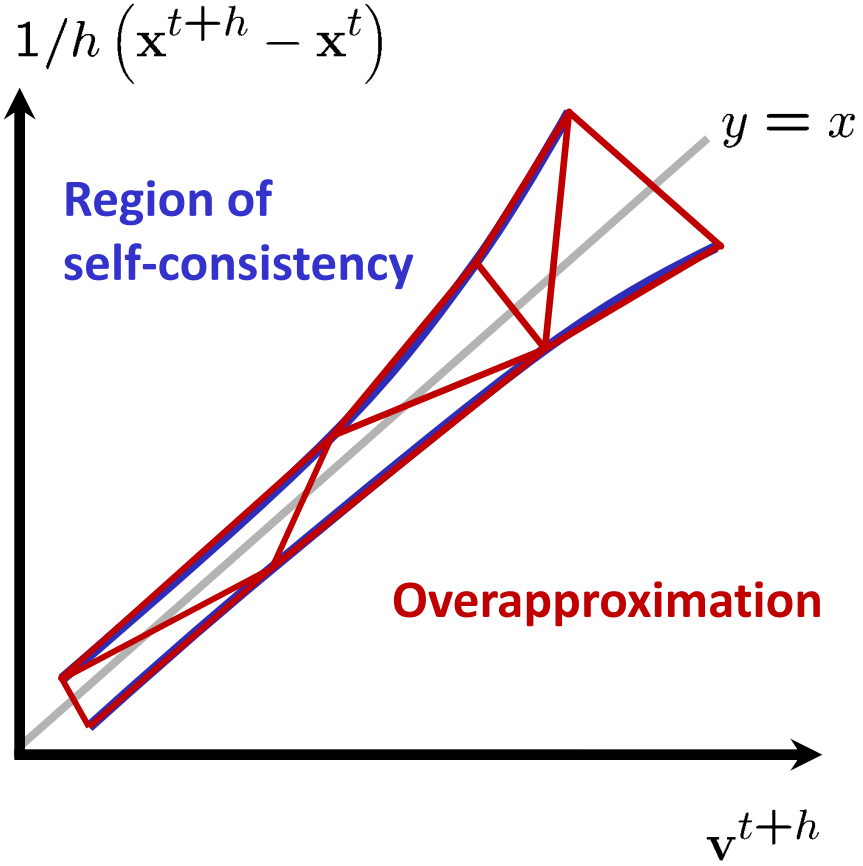
- Model-based consistency checks
- Model vs. vehicle state
- Map-based path validation
- Exception signal if inconsistency threshold is exceeded



High Assurance Code/Proof Co-Synthesis



Detection Through Feasible Region of State



Self-consistency equation

$$\mathcal{F}:\begin{pmatrix}\mathbf{x}^{t}\\ \mathbf{v}^{t}\\ \mathbf{x}^{t+h}\\ \mathbf{v}^{t+h}\end{pmatrix}\mapsto\begin{pmatrix}1/h\left(\mathbf{x}^{t+h}-\mathbf{x}^{t}\right)\\ \mathbf{v}^{t+h}\end{pmatrix}$$

Inside a polyhedra

$$A_i\mathcal{F}(\vec{x})-b_i\preceq\vec{0}$$

Test: attack-free, if $\mathcal{F}\left(\mathbf{s}^{t}\oplus\mathbf{s}^{t+h}\right)\in\bigcup_i\mathcal{P}_i$

Rule-Based Formal Compilation Flow

HCOL specification

$$\mathrm{ForAny}_{i=0}^{k-1}\left(\mathrm{InsidePoly}_{m,n}(\mathbf{A}_i, \mathbf{b}_i, .)\right)$$

Expansion + backtracking

HCOL (dataflow) expression

$$\mathrm{Reduction}_{n,(a,b)\mapsto a\vee b} \circ \left[\cdot\right]_{\ell=0}^{k-1}\Big(\mathrm{Reduction}_{n,(a,b)\mapsto a\wedge b} \circ \mathrm{Pointwise}_{n,x_i\mapsto x_i\leq 0} \circ \mathrm{Pointwise}_{n,x_i\mapsto x_i-b_i} \circ \left[\cdot\right]_{i=0}^{m-1}\left(\mathrm{Reduction}_{n,(a,b)\mapsto a+b} \circ \mathrm{Pointwise}_{n,x_j\mapsto a_{i,j}x_j}\right)\Big)$$

Recursive descent

Σ-HCOL (loop) expression

$$\mathrm{Reduction}_{n,(a,b)\mapsto a\vee b} \circ \sum_{\ell=0}^{k-1}\!\!\cup\Big(\mathrm{e}_\ell^k \circ \mathrm{Reduction}_{n,(a,b)\mapsto a\wedge b} \circ \mathrm{Pointwise}_{n,x_i\mapsto x_i\leq 0} \circ \mathrm{Pointwise}_{n,x_i\mapsto x_i-b_i} \circ \sum_{i=0}^{m-1}\!\!\cup\Big(\mathrm{e}_i^m \circ \mathrm{Reduction}_{n,(a,b)\mapsto a+b} \circ \sum_{j=0}^{n-1}\!\!\cup\Big(\mathrm{e}_j^n \circ \mathrm{Atomic}_{x_j\mapsto a_{i,j}x_j} \circ (\mathrm{e}_j^n)^\top\Big)\Big)\Big)$$

Confluent term rewriting

Optimized Σ-HCOL expression

$$\sum_{\ell=0}^{k-1}\!\!\vee\Big(\sum_{i=0}^{m-1}\!\!\wedge\Big(\mathrm{Atomic}_{x\mapsto(x-b_i)\leq 0} \sum_{j=0}^{n-1}\!\!+\Big(\mathrm{Atomic}_{x\mapsto a_{i,j}x} \circ (\mathrm{e}_j^i)\top\Big)\Big)\Big)$$

Recursive descent

Abstract code

```
func(TVoid, "transform", [ Y, X ],
       data(D1, V([ V([ V([ V(1.0), V(2.0) ]), V([ V(3.0), V(4.0) ]), V([ V(5.0), V(
          data(D2, V([ V([ V(1.0), V(2.0), V(3.0) ]), V([ V(1.0), V(2.0), V(3.0) ])
             chain(
                assign(nth(Y, V(0)), V(false)),
                loopc(i1, [ 0 .. 1 ], neq(nth(Y, V(0)), V(true)),
                   decl([ s8 ],
                      chain(
                         assign(s8, V(true)),
                         loopc(i19, [ 0 .. 2 ], neq(s8, V(false)),
                            decl([ s10, s11, s12, s13, s14, s9 ],
```

extends [PLDI 2005, DSLWC 2009]

HCOL: Hybrid Control Operator Language

Sensor values and model-based predictions

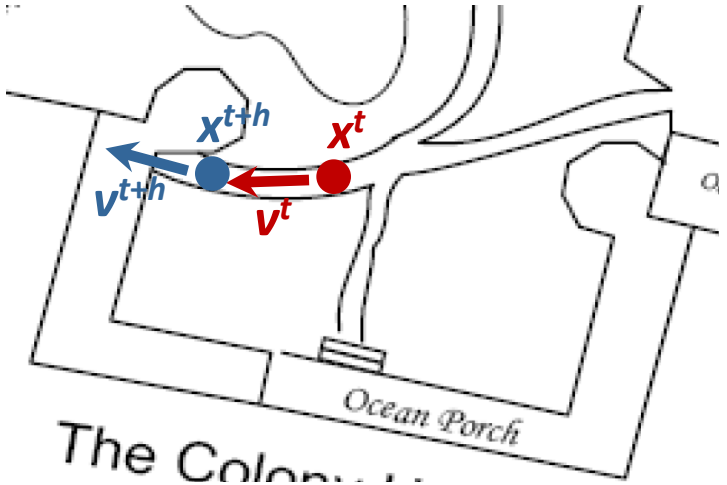
Euler step: x^{t+h}

$$\mathbf{x}^{t+h} \approx \left[\mathrm{I}_3 \,|h\,\mathrm{I}_3\right](\mathbf{x}^t \oplus \mathbf{v}^{t+h})$$

Numerical differentiation: v^{t+h}

$$\mathbf{v}^{t+h} \approx 1/h\left[\mathrm{I}_3 \,|-\mathrm{I}_3\right](\mathbf{x}^{t+h} \oplus \mathbf{x}^t)$$

I_3: 3 x 3 identity matrix
time step = matrix-vector product



Assurance through guaranteed controller input and output

- Declarative representation of physics, data and control algorithms
- Enables rule-based software synthesis and variant generation, verification and proof co-synthesis
- Extends Spiral's OL and SPL languages into the control domain

HCOL: Control Operator Examples

Time step residue: Disagreement between model and sensors

$$\mathbf{r}^{t+h} = \mathbf{R} \cdot (\mathbf{x}^t \oplus \mathbf{v}^t \oplus \mathbf{x}^{t+h} \oplus \mathbf{v}^{t+h})$$

$$\mathbf{R} = \begin{bmatrix} -\mathrm{I}_3 & h\,\mathrm{I}_3 & \mathrm{I}_3 & 0_3 \\ 1/h\,\mathrm{I}_3 & 0_3 & 1/h\,\mathrm{I}_3 & \mathrm{I}_3 \end{bmatrix}$$

Error operator: L_2 norm of time step residue

$$\mathsf{E}_h : (\mathbf{s}^t, \mathbf{s}^{t+h}) \mapsto (\mathbf{s}^t \oplus \mathbf{s}^{t+h})^\top \left(\mathbf{R}^\top \mathbf{R}\right) (\mathbf{s}^t \oplus \mathbf{s}^{t+h})$$

$$\mathbf{s}^t = (\mathbf{x}^t \oplus \mathbf{v}^t), \quad \mathbf{s}^{t+h} = (\mathbf{x}^{t+h} \oplus \mathbf{v}^{t+h})$$

PID controller: Control velocity at set point v_0

$$\begin{pmatrix} \mathbf{u}^t \\ \mathbf{s}^t \end{pmatrix} = \left(\begin{bmatrix} k_p\,\mathrm{I}_e & k_i\,\mathrm{I}_3 & k_d/h\,\mathrm{I}_3 \end{bmatrix} \begin{bmatrix} \mathrm{I}_3 & . & . \\ \mathrm{I}_3 & . & \mathrm{I}_3 \\ \mathrm{I}_3 & -\mathrm{I}_3 & . \end{bmatrix} \begin{bmatrix} \mathrm{I}_3 & -\mathrm{I}_3 & . \\ . & . & \mathrm{I}_6 \end{bmatrix} \right) \oplus \begin{bmatrix} \mathrm{I}_3 & -\mathrm{I}_3 & . \\ \mathrm{I}_3 & -\mathrm{I}_3 & \mathrm{I}_3 \end{bmatrix} (\mathbf{v}_0 \oplus \mathbf{v}^t \oplus \mathbf{s}^{t-h})$$

$$\mathbf{e}^t = \mathbf{v}^0 - \mathbf{v}^t, \quad \mathbf{s}^t = \mathbf{e}^t \oplus \sum_{i=0}^{n-1} \mathbf{e}^{ih}$$

Usual PID controller definition: $\mathbf{u}^t = k_p \mathbf{e}^t + k_i \sum_{i=0}^{n-1} \mathbf{e}^{ih} + k_d \dfrac{\mathbf{e}^t - \mathbf{e}^{t-h}}{h}$

Rule-Based Code Synthesis

High Level Rules: Transformations within high level abstraction

$$\mathrm{I}_n \rightarrow \sum_{i=0}^{n-1} \mathrm{e}_i^n \, \mathrm{I}_1 (\mathrm{e}_i^n)^\top$$

$$\left[\sum_{i=0}^{n-1} \mathsf{S}_i \, A_i \, \mathsf{G}_i \,|\, \sum_{i=0}^{n-1} \mathsf{S}_i \, B_i \, \mathsf{G}_i \right] \rightarrow \sum_{i=0}^{n-1} \mathsf{S}_i \left(\left[A_i | B_i \right] \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \mathsf{G}_i$$

with $\mathrm{e}_i^{1 \times n} = [0, \ldots, 0, 1, 0, \ldots, 0]$

Code generation rules: Translate high level abstraction into code

$$\begin{aligned}
\mathrm{Code}\left(y = (A\,B)x\right) &\rightarrow \left\{ \mathtt{Decl}(t), \mathrm{Code}\left(t = Bx\right), \mathrm{Code}\left(y = At\right) \right\} \\
\mathrm{Code}\left(y = \left(\sum_{i=0}^{n-1} A_i \right) x \right) &\rightarrow \left\{ y := \vec{0}, \mathtt{for}(i = 0..n-1)\ \mathrm{Code}\left(y \mathrel{+}= A_i x\right) \right\} \\
\mathrm{Code}\left(y = \mathrm{e}_i^{1 \times n} x\right) &\rightarrow y[0] := x[i] \\
\mathrm{Code}\left(y = \mathrm{e}_i^{n \times 1} x\right) &\rightarrow \left\{ y = \vec{0}, y[i] := x[0] \right\}
\end{aligned}$$

Symbolic Rule Verification

- **Rule replaces left-hand side by right-hand side when preconditions match**

$$\mathrm{I}_n \rightarrow \sum_{i=0}^{n-1} \mathrm{e}_i^n \, \mathrm{I}_1 (\mathrm{e}_i^n)^\top$$

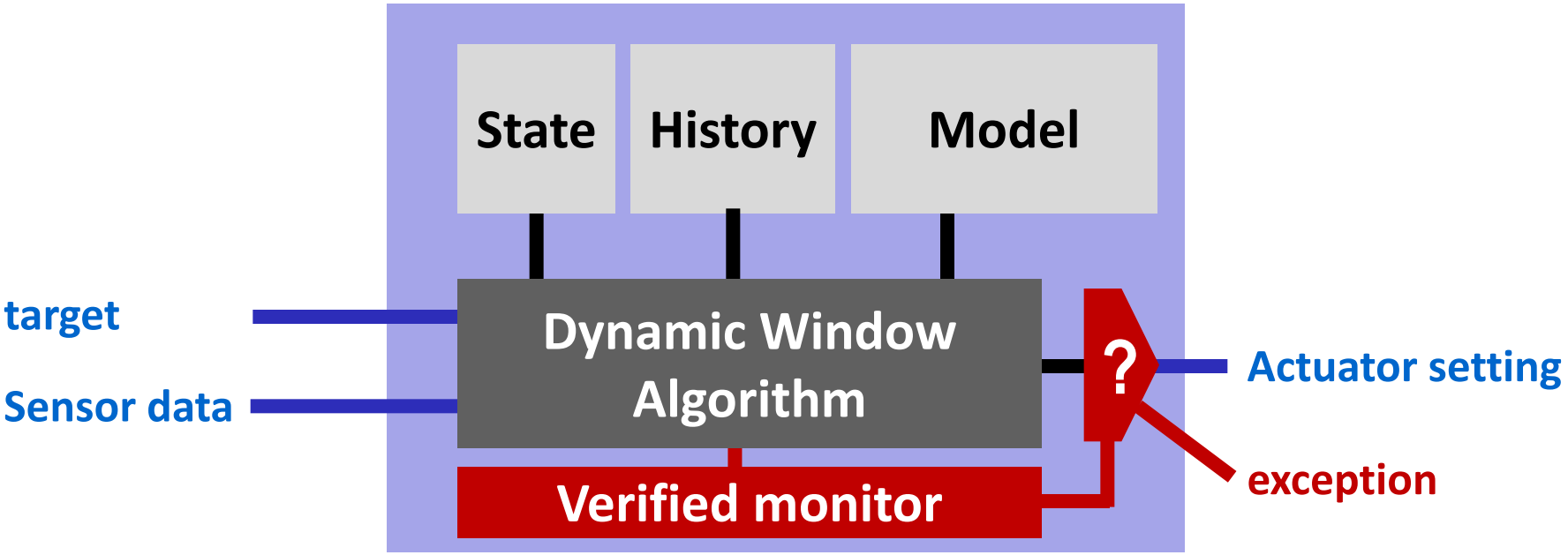
- **Test rule by symbolically evaluating expressions before and after rule application and compare result**

$$\mathrm{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad = ? \quad \sum_{i=0}^{2} \mathrm{e}_i^3 \, \mathrm{I}_1 (\mathrm{e}_i^3)^\top = [1\,0\,0][1][1\,0\,0]^\top + [0\,1\,0][1][0\,1\,0]^\top + [0\,0\,1][1][0\,0\,1]^\top$$

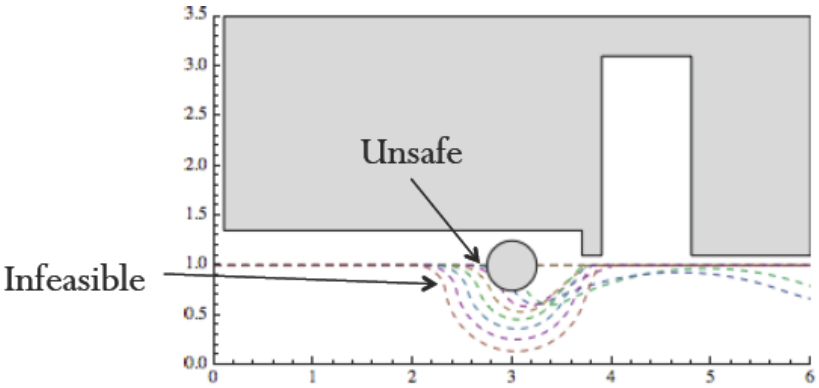
Algorithms Formalized in HA Spiral So Far

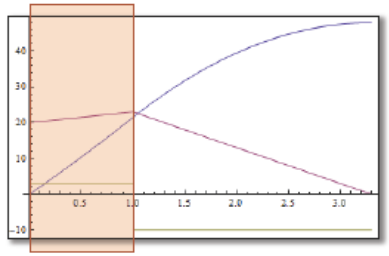
- **Dynamic Window Approach Monitor**
  Passive safety monitor, formally derived in KeYmaera
- **Set calculus: Sensor self-consistency in state space**
  Check that set of self-consistent true state values permitted by measurements is non-empty
- **Multi-timescale Z-test for redundant sensors**
  Test for zero mean of difference between multiple sensors on multiple time scales
- **Mathematical infrastructure robotics code**
  Coordinate transformations, data filtering, ODE integration

Example: Dynamic Window Safety Monitor



Dynamic Window Approach Primer





$$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r+V)\right)$$

Algorithm Verified in KeYmaera

Theorem and proof

$$\textbf{To Prove: } \psi_{ps} \rightarrow [dw_{ps}]\left((v_r = 0) \vee \left(\|p_r - p_o\| > \frac{v_r^2}{2b} + V\frac{v_r}{b}\right)\right)$$

$$dw_{ps} \equiv (ctrl_o \;||\; ctrl_r;\; dyn)^*$$

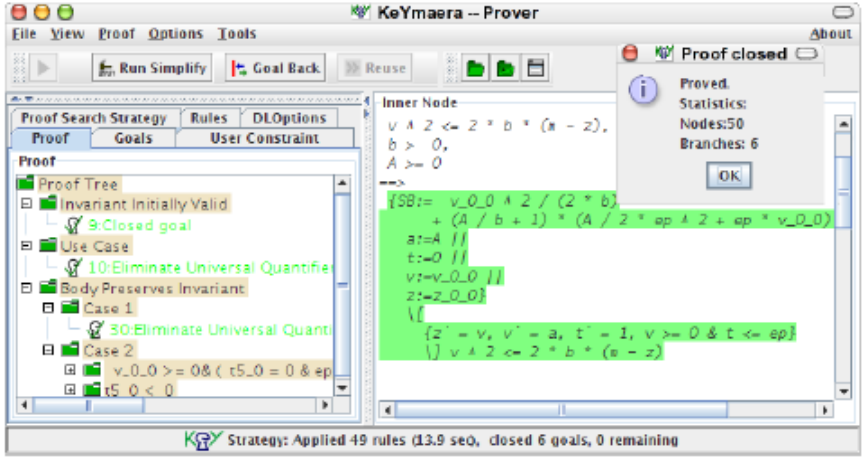
$$ctrl_o \equiv v_o = (*,*);\; ?\|v_o\| \leq V$$

$$ctrl_r \equiv (a_r := -b) \cup (?v_r = 0;\; a_r := 0;\; \omega_r := 0) \cup (a_r := *;\; ?-b \leq a_r \leq A;\; \omega_r := *;\; ? -\Omega \leq \omega_r \leq \Omega;\; p_c := (*,*);\; d_r := (*,*);\; p_o := (*,*);\; ?feasible \wedge safe)$$

$$feasible \equiv \|p_r - p_c\| > 0 \wedge \omega_r \|p_r - p_c\| = v_r \wedge d_r = \frac{(p_r - p_c)^\perp}{\|p_r - p_c\|}$$

$$safe \equiv \|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$$

$$dyn \equiv (t := 0;\; p_r^{x\prime} = v_r d_r^x,\; p_r^{y\prime} = v_r d_r^y,\; d_r^{x\prime} = -\omega_r d_r^y,\; d_r^{y\prime} = \omega_r d_r^x,\; p_o^{x\prime} = v_o^x,\; p_o^{y\prime} = v_o^y,\; v_r' = a_r,\; \omega_r' = \frac{a_r}{\|p_r - p_c\|},\; t' = 1 \;\&\; v_r \geq 0 \wedge t \leq \varepsilon)$$



Resulting safety monitor condition

$$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$$

	$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon v_r\right)$
	$v_r = 0 \vee \|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$
Sensor uncertainty	$\|\tilde{p}_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right) + U_p$
Actuator disturbance	$\|p_r - p_o\|_\infty > \frac{v_r^2}{2bU_m} + V\frac{v_r}{bU_m} + \left(\frac{A}{bU_m}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$
Sensor failure	$\|\tilde{p}_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v + V)\right) + U_p + g\Delta$
	$v_r = 0 \vee \|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + \frac{V^2}{2b_o} + V\left(\frac{v_r}{b} + \tau\right) + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$

Mathematical Objects in HCOL

- **Infinity norm**

$$\|.\|_\infty^n : \mathbb{R}^n \to \mathbb{R}$$
$$(x_i)_{i=0,\ldots,n-1} \mapsto \max_{i=0,\ldots,n-1} |x_i|$$

- **Chebyshev distance**

$$d_\infty^n(.,.) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$(x, y) \mapsto \|x - y\|_\infty^n$$

- **Vector subtraction**

$$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

Implicit Isomorphisms

$$\mathbb{R}^m \times \mathbb{R}^n \cong \mathbb{R}^{m+n}$$
$$\mathbb{R} \cong \mathbb{R}^1$$
$$\mathbb{R}^k \times (\mathbb{R}^m \times \mathbb{R}^n) \cong \mathbb{R}^k \times \mathbb{R}^m \times \mathbb{R}^n$$
$$(\mathbb{R}^k \times \mathbb{R}^m) \times \mathbb{R}^n \cong \mathbb{R}^k \times \mathbb{R}^m \times \mathbb{R}^n$$

- **Pointwise comparison**

$$(<)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{Z}_2^n$$
$$\left((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\right) \mapsto (x_i < y_i)_{i=0,\ldots,n-1}$$

HCOL Basic Operators

$$\text{Pointwise}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}^n$$
$$(x_i)_i \mapsto f_0(x_0) \oplus \cdots \oplus f_{n-1}(x_{n-1})$$

$$\text{Pointwise}_{n\times n,f_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$\Big((x_i)_i, (y_i)_i\Big) \mapsto f_0(x_0, y_0) \oplus \cdots \oplus f_{n-1}(x_{n-1}, y_{n-1})$$

$$\text{Reduction}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}$$
$$(x_i)_i \mapsto f_{n-1}(x_{n-1}, f_{n-2}(x_{n-2}, f_{n-3}(\ldots f_0(x_0, \text{id}())\ldots)$$

$$\text{Induction}_{n,f_i} : \mathbb{R} \to \mathbb{R}^n$$
$$x \mapsto (\text{id}, f_1(x, \text{id}), f_2(x, f_1(x, \text{id})), \ldots)^\top$$

$$\text{Atomic}_{f(.)} : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto f(x)$$

$$\text{Atomic}_{f(.,.)} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$
$$(x, y) \mapsto f(x, y)$$

$$\text{Scale}_{n,(a,b)\mapsto a\diamond b} : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$$
$$\Big(\alpha, (x_i)_{i=0,\ldots,n-1}\Big) \mapsto (\alpha \diamond x_i)_{i=0,\ldots,n-1}$$

Formal Code Synthesis

- **HCOL Breakdown Rules**

$$\text{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \rightarrow \Big(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\Big)(.,.,.)$$

$$\text{with} \quad a_0 = \tfrac{1}{2b},\ a_1 = \tfrac{V}{b} + \varepsilon\Big(\tfrac{A}{b}+1\Big),\ a_2 = \Big(\tfrac{A}{b}+1\Big)\Big(\tfrac{A}{2}\varepsilon^2 + \varepsilon V\Big)$$

$$d_\infty^n(.,.) \rightarrow \|.\|_\infty^n \circ (-)_n$$

$$(\diamond)_n \rightarrow \text{Pointwise}_{n\times n,(a,b)\mapsto a\diamond b}$$

$$\|.\|_\infty^n \rightarrow \text{Reduction}_{n,(a,b)\mapsto \max(|a|,|b|)}$$

$$< .,. >_n \rightarrow \text{Reduction}_{n,(a,b)\mapsto a+b} \circ \text{Pointwise}_{n\times n,(a,b)\mapsto ab}$$

$$P[x,(a_0,\ldots,a_n)] \rightarrow < (a_0,\ldots,a_n),. > \circ (x^i)_n$$

$$(x^i)_n \rightarrow \text{Induction}_{n,(a,b)\mapsto ab,1}$$

- **Fully Expanded HCOL Expression**

$$\text{SafeDist}_{V,A,b,\varepsilon} \rightarrow \text{Atomic}_{(x,y)\mapsto x<y} \circ \Big(\Big(\text{Reduction}_{3,(x,y)\mapsto x+y} \circ \text{Pointwise}_{3,x\mapsto a_i x} \circ \text{Induction}_{3,(a,b)\mapsto ab,1}\Big) \times \Big(\text{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \text{Pointwise}_{2\times 2,(x,y)\mapsto x-y}\Big)\Big)$$

HCOL and Σ–HCOL

- **HCOL problem specification:** *declarative, point free*

$$\text{ForAny}_{i=0}^{k-1}\left(\text{InsidePoly}_{m,n}(\mathbf{A}_i, \mathbf{b}_i, .)\right)$$

- **HCOL expression:** *data flow graph*

$$\text{Reduction}_{n,(a,b)\mapsto a\vee b} \circ \left[\begin{matrix}\cdot\\ \cdot\end{matrix}\right]_{\ell=0}^{k-1} \Big(\text{Reduction}_{n,(a,b)\mapsto a\wedge b} \circ \text{Pointwise}_{n,x_i\mapsto x_i\leq 0} \circ \text{Pointwise}_{n,x_i\mapsto x_i-b_i} \circ \left[\begin{matrix}\cdot\\ \cdot\end{matrix}\right]_{i=0}^{m-1} \left(\text{Reduction}_{n,(a,b)\mapsto a+b} \circ \text{Pointwise}_{n,x_j\mapsto a_{i,j}x_j}\right)\Big)$$

- **Σ-HCOL expression:** *optimized loop structure*

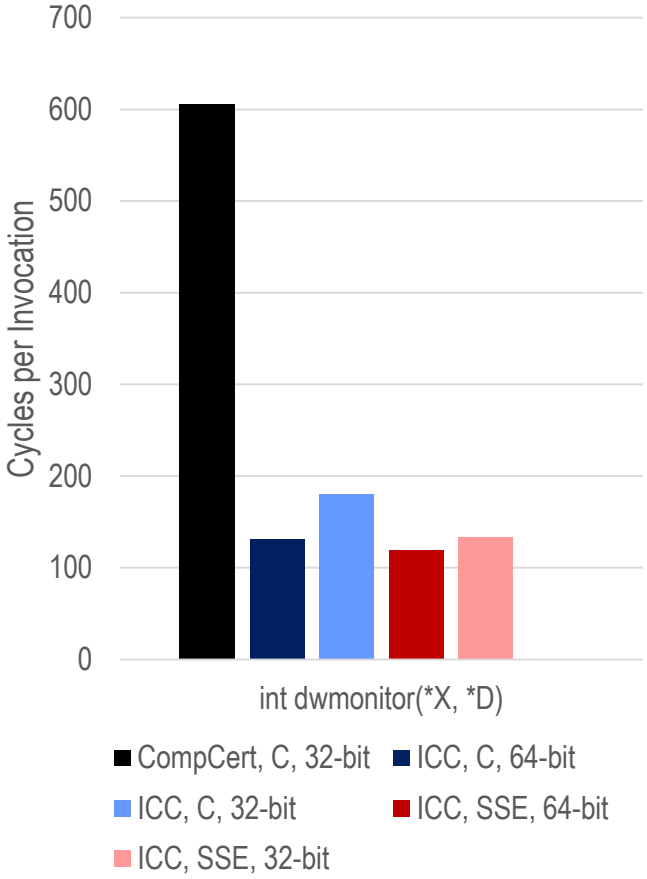
$$\sum_{\ell=0}^{k-1}\mathbb{V}\left(\sum_{i=0}^{m-1}\mathbb{\wedge}\left(\text{Atomic}_{x\mapsto(x-b_i)\leq 0}\sum_{j=0}^{n-1}\mathbb{+}\left(\text{Atomic}_{x\mapsto a_{i,j}x}\circ(\mathrm{e}_j^i)\top\right)\right)\right)$$

Final Synthesized Interval Arithmetic Code

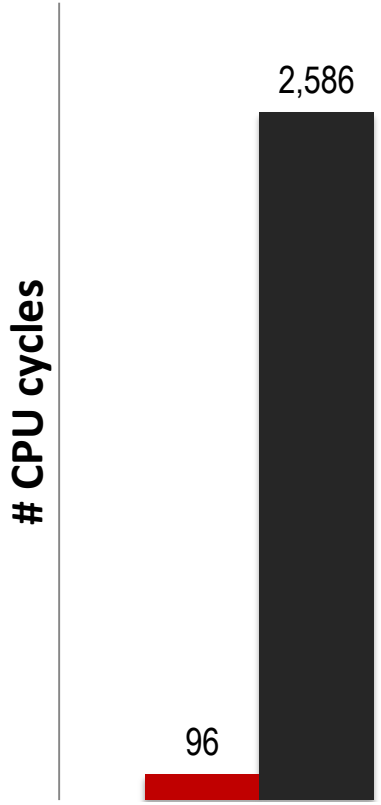
```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17, x18, x19, x2, x3, x4, x6, x7, x8, x9;
    int w1;
    {
        unsigned _xm = _mm_getcsr();
        _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
        u5 = _mm_set1_pd(0.0);
        u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
        u1 = _mm_set_pd(1.0, (-1.0));
        for(int i5 = 0; i5 <= 2; i5++) {
            x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
            x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
            x2 = _mm_mul_pd(x1, x6);
            x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)), x6);
            x4 = _mm_sub_pd(_mm_set1_pd(0.0), _mm_min_pd(x3, x2));
            u3 = _mm_add_pd(_mm_max_pd(_mm_shuffle_pd(x4, x4, _MM_SHUFFLE2(0, 1)), _mm_max_pd(x3, x2)), _mm_set1_
            u5 = _mm_add_pd(u5, u3);
            x7 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
            x8 = _mm_mul_pd(x7, u2);
            x9 = _mm_mul_pd(_mm_shuffle_pd(x7, x7, _MM_SHUFFLE2(0, 1)), u2);
            x10 = _mm_sub_pd(_mm_set1_pd(0.0), _mm_min_pd(x9, x8));
            u1 = _mm_add_pd(_mm_max_pd(_mm_shuffle_pd(x10, x10, _MM_SHUFFLE2(0, 1)), _mm_max_pd(x9, x8)), _mm_set
        }
        u6 = _mm_set1_pd(0.0);
        for(int i3 = 0; i3 <= 1; i3++) {
            u8 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(i3 + 1)])));
            u7 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(3 + i3)])));
            x14 = _mm_add_pd(u8, _mm_shuffle_pd(u7, u7, _MM_SHUFFLE2(0, 1)));
            x13 = _mm_shuffle_pd(x14, x14, _MM_SHUFFLE2(0, 1));
            u4 = _mm_shuffle_pd(_mm_min_pd(x14, x13), _mm_max_pd(x14, x13), _MM_SHUFFLE2(1, 0));
            u6 = _mm_shuffle_pd(_mm_min_pd(u6, u4), _mm_max_pd(u6, u4), _MM_SHUFFLE2(1, 0));
        }
        x17 = _mm_addsub_pd(_mm_set1_pd(0.0), u6);
        x18 = _mm_addsub_pd(_mm_set1_pd(0.0), u5);
        x19 = _mm_cmpge_pd(x17, _mm_shuffle_pd(x18, x18, _MM_SHUFFLE2(0, 1)));
        w1 = (_mm_testc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff
             (_mm_testnzc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffff
```

Spiral Interval Arithmetic Code Quality

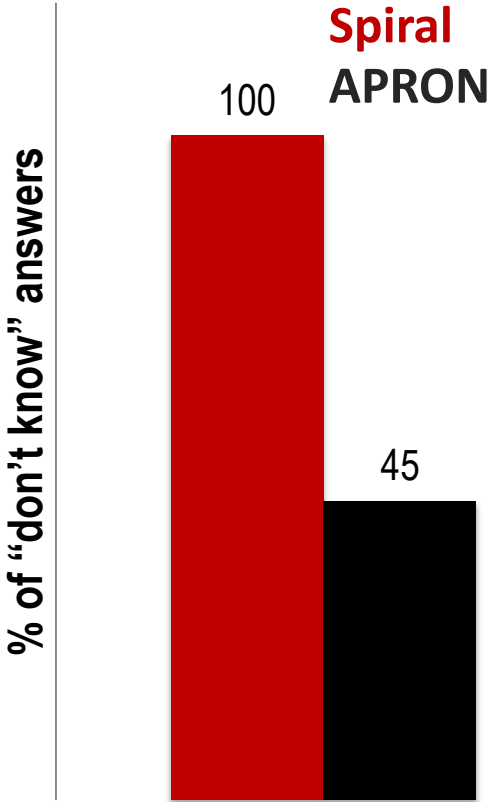
Intel C vs. CompCert



Performance



Precision at boundary



$a < b$? for
$a - b \approx 10^{-15}$

SandyBridge CPU, Intel C Compiler, CompCert, APRON Interval Arithmetic Library

High Assurance Spiral In A Nutshell

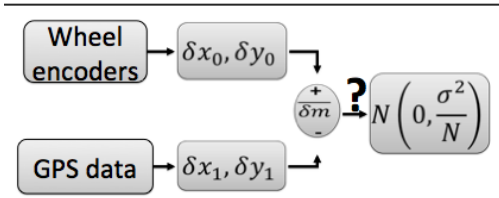
Problem and main idea

Co-synthesize high-quality code and proof for sensor-fusion based self-consistency algorithms

Deployed in Landshark robot

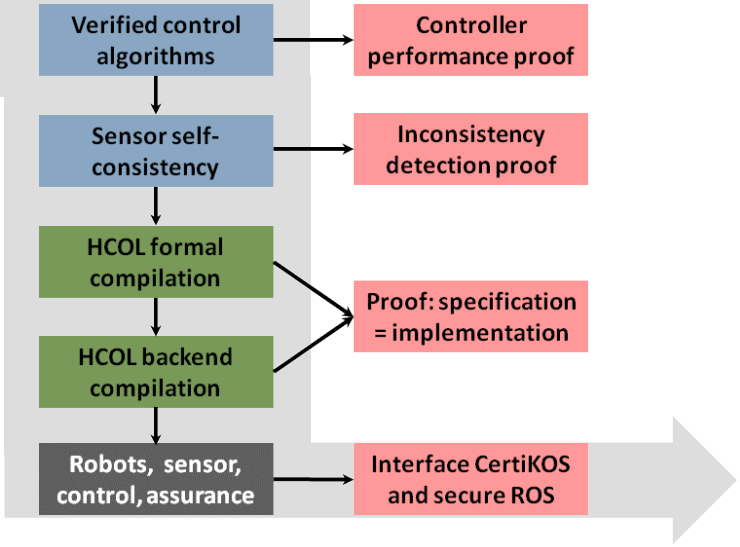
Approach

$$\mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{Z}_2$$
$$(v_r, p_r, p_o) \mapsto \left(p(v_r) < d_\infty(p_r, p_o)\right)$$



$$A_i \mathcal{F}(\vec{x}) - b_i \preceq \vec{0}$$

$$\mathbf{x}^{t+h} \approx \left[\mathrm{I}_2 \,|h\, \mathrm{I}_2\right](\mathbf{x}^t \oplus \mathbf{v}^{t+h})$$




```
int dwmonitor(float  *X, double  *D)
    __m128d u1, u2, u3, u4, u5, u6,
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(
    _mm_set1_ps(FLT_MIN), _mm_set1_p
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_
            +DBL_MIN)), _mm_loaddup_
        x1 = _mm_addsub_pd(_mm_set1_
        x2 = _mm_mul_pd(x1, x6);
    ...
        }
    __asm nop;
    if (_mm_getcsr() & 0x0d) {
        _mm_setcsr(_xm);
```

More Information:

www.spiral.net

www.spiralgen.com